

THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
DIPLOMA IN EDUCATION EXAMINATION



741

MATHEMATICS

Time: 3 Hours

Monday, May 09, 2005 p.m.

Instructions

1. This paper consists of sections A and B.
2. Answer *all* questions in section A and *three (3)* questions from section B.
3. Section A carries 49 marks and section B carries 51 marks.
4. Show your workings and answers clearly in both sections.
5. Graph papers, mathematical and statistical tables may be used.
6. Cellular phones are *not* allowed in the examination room.
7. Write your *Examination Number* on every page of your answer booklet(s).

ICC

This paper consists of 4 printed pages.

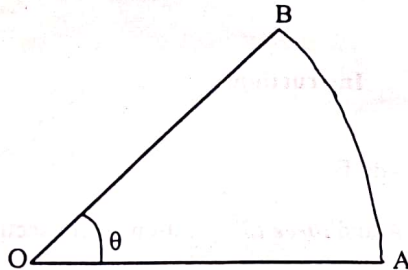
SECTION A (49 marks)

Answer all questions in this section.

1. Find the differential coefficient of $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$

✓ 2. Solve for x if $\sqrt{2x-1} - \sqrt{x-1} = 1$

3. The angle of a sector AOB shown in the diagram below is θ radius and the radius of the circle is r cm. The area of the sector is 10 cm^2 and its total perimeter is 13 cm . Find r and θ .



- ✓ 4. Simplify as far as possible.

$$\frac{(a^{2r-3} b^{-1-s} a^{s-r})^t \cdot (ab^{-1})^t}{(a^{-rt+2t} b^{2t})^{-1}}$$

✓ 5. (a) $\int \cos^2 x \sin^3 x \, dx$.

(b) Express e^i in the form $x + iy$.

- ✓ 6. Solve the following linear systems by using determinants:

$$-2x_1 + 3x_2 - x_3 = 1$$

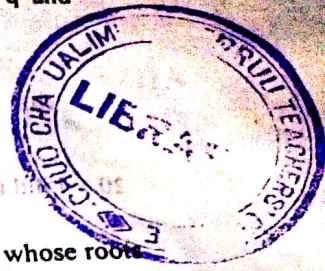
$$x_1 + 2x_2 - x_3 = 4$$

$$-2x_1 - x_2 + x_3 = -3$$

✓ 7. Show that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$.

8. If $a + ib = \frac{1}{x + iy}$; show that $(a^2 + b^2)(x^2 + y^2) = 1$.

9. A box contains 20 fuses, of which 5 are defective. If 3 of the fuses are selected at random and removed from the box in succession without replacement, what is the probability that all three fuses are defective?



10. Find the volume V of a triangular pyramid $ABCD$ with vertices $A(2, -1, 1)$; $B(5, 5, 4)$; $C(3, 2, -1)$; and $D(4, 1, 3)$.
11. Determine the three geometric progression such that their sum is 52; given the condition that the sum of their product taken in pairs is 624.
- ✓ 12. If one root of the equation $8x^2 + qx + 27 = 0$ is the square of the other, find the value of q and hence rewrite the equation.
- ✓ 13. If $\log_{10} 2 = 0.3010300$ and $\log_{10} 3 = 0.4771213$, without using tables, find the values of
- (a) $\log_{10} 50$ to 7 decimal places.
- ✓ (b) $\log_{10} 13.5$ to 7 decimal places.
- ✓ 14. (a) If $p + q = 5$ and $p^2 + q^2 = 19$, find the value of pq and write the equation in x whose roots are p and q .
- (b) A geometric series with common ratio 0.8 converges to the sum 250. Find the fourth term of the series.
15. (a) If $y = (\sin^{-1} x)^2$, show that $(1 - x^2) \left(\frac{dy}{dx} \right)^2 = 4y$.
- (b) Find the equations of the tangent and normal to the curve $y = (1 - x)(2 + x)$ at the point where $x = 2$.

SECTION B (51 marks)

Answer **three (3)** questions from this section.

- ✓ 16. (a) If $Z + \frac{1}{Z} = -1$, prove that $Z^5 + \frac{1}{Z^5} = -1$ and find the value of $Z^{11} + \frac{1}{Z^{11}}$.
- ✓ (a) Prove that $\cos^6 \theta = \frac{1}{32} (\cos 6\theta + 6 \cos 4\theta + 15 \cos 2\theta + 10)$.
17. (a) To estimate the time mean (μ) weight of all the automobiles that use a certain large parking garage; a statistician takes a random sample of size 100, obtaining a sample mean $\bar{x} = 2030$ kilograms and a sample standard deviation 216 kilograms. What is the 95 % confidence interval estimate for μ ?
- (b) Two students were informed that they received standard deviation scores of 0.8 and -0.4 respectively on a multiple choice examination in English language. If their marks were 88 and 64 respectively, find the mean and standard deviation of the examination.
- ✓ 18. (a) Find a general solution to the second order differential equation given by $XY^{11} = 2$.
- ✓ (b) Solve for initial value problem:
- $(x^2 + 1)y'' + (y^2 + 1) = 0$ given $y(0) = 1$.

(c) If $Z = \sin^{-1} \left(\frac{x}{x} - \frac{y}{y} \right)$, prove that $x \frac{\delta z}{\delta x} + y \frac{\delta z}{\delta y} = 0$.

✓ 19. (a) Evaluate the following :

$$I = \int_1^2 \int_0^3 \int_0^1 (p^2 + q^2 - r^2) dp dq dr.$$

(b) Using Wallis' formula, evaluate this equation: $I_6 = \int_0^{\frac{\pi}{2}} \sin^6 x \, dx$.

20. Find the local extrema for: $f(x, y) = x^3 + y^3 - 6xy$.